HW #5 Quadrature

Name:JianWang FSUID:JW09R

**0. Executive Summary**

Write adaptive quadrature which uses simpson’s rule to calculate Gaussian probability function and obtain the result when the x is equal to 1.0 (at least six significant digits). Besides, use the function defined above to calculate the result of the call option via the B-S formula(in four digit numbers). Finally, I use bisection method to find out the implied volatility (in four significant digits). The result are listed as follows:

The adaptive quadrature function that use the simpson’s rule is working correctly.

The integral of Gaussian probability function from minus infinity to 1 is 0.841345

The calculated Call option price is 98.4794

The calculated implied volatility is 7.609%

If I use the Tol equal to, then the related error will be less than 5\*

1. **Statement of Problem**

To find the value of the call option price, we should use the adaptive quadrature method to calculate the Gaussian probability which can not derive the value of the integral directly. Besides, show the algorithm is working correctly. In the end, to find the volatility of the BS model given the assumptions.

1. **Description of The Mathematics**

The two order of quadrature polynomialis:



Where, 

From this we can get the approximation of the integral as follows:

where 

With the help of Taylor’s expansion , we can get the Error E, the process is as follows:



  such that:



We also know that , for the simpson’s method the r=4 and the m=4



 when 

So, when , where  is a constant

So if we half the interval each time, and let,  keep the values of two rounds calculation. 

In the adaptive method, we use  instead of. Then we got the formula as below:



So we can make sure the total error less than given tolerance by checking 

1. **Description of the Algorithm**
2. The simpson’s rule

Suppose we have f be integrated function, [a,b] be the closed interval, then the simpson’s rule is that:

**Input** f,a,b

Return (f(a)+f(a+(b-a)/2)\*4+f(b))\*(b-a)/6

END

1. Adaptive algorithm

Suppose we have f be integrated function, [a,b] be the closed interval, Tol be the given absolute error.

The quadrature rule is

**Input** f, a, b, Tol

Ic = Q(f, a, b)

If =Q (f, a, (a+b)/2) + Q (f, (a+b)/2, b)

E = (If-Ic)/15

IF abs(E) > Tol

IL=adaptive\_Q ( f, a, (a+b)/2, Tol/2 )

IR=adaptive\_Q ( f, (a+b)/2, b, Tol/2)

Return IL+IR

Otherwise

Return If+E

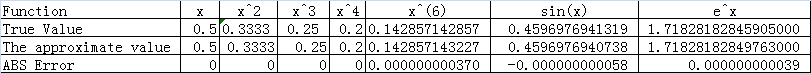
End if

END

Since we only need at least 6 significant digits, so I used the as the Tol.

To show the algorithm is working correctly, I test the routine on some integrals that I know the exact value. The interval of the integration is [0,1]

The result is as follows:



From the above result, we can see that since the error polynomial is of 4th order, so the error of the first three functions are Zero. Besides the abs error of the other functions are very small, which showed that the routing can work well to be the approximation of the integration.

1. **Results**
   1. The integral of Gaussian probability function from minus infinity to 1 is 0.841345 with six significant numbers.

I integrate the function from 0 to 1 and the result is 0.341345.According to the property of symmetric, the Gaussian probability function integrated from minus infinity to 0 is 0.5. so the probability from minus infinity to 1 is 0.841345, with six significant digits.

Since the absolute error is bounded by the parameter Tol which is chosen as 10E-7, and P>0.5, so the relative error  and the answer has six significant digits.

* 1. The calculated Call option price is 98.4794 with four significant numbers.

The process is as follows:

With the formula: and the given assumptions,

S= 3441.0

K=3425.0

r=6.75%

T=54/365

t=0

delta=0.1350

I calculated the result as follows:

|  |  |
| --- | --- |
| d1 | 0.25726166 |
| d2 | 0.20533574 |
| P(d1) | 0.60151161 |
| P(d2) | 0.58134509 |

I also check the result with the table of the standard normal distribution, the result is similar.

So the call price is 98.4794

Besides, 

Which answer the figures that I think I shall have

* 1. The calculated implied volatility is 7.609% with four significant numbers.

To calculate the implied volatility, I used bisection method which was written before and the volatility rate is 7.609% with 4 significant numbers. The assumption are as follows:

S= 3441.0

K=3425.0

r=6.75%

T=54/365

t=0

c=94

1. **Conclusions**

My conclusions are as follows:

The adaptive quadrature function that use the simpson’s rule is working correctly.

The integral of Gaussian probability function from minus infinity to 1 is 0.841345

The calculated Call option price is 98.4794

The calculated implied volatility is 7.609%

If I use the Tol equal to, then the related error will be less than 5\*

1. **Code Listing**

Private Sub simpson()

Dim a As Double

Dim b As Double

Dim tol As Double

a = 0

b = 0.25726165943415

tol = 10 ^ (-7)

Worksheets("sheet1").Range("a1").Value = simp(a, b)

Worksheets("sheet1").Range("a2").Value = adaptive(a, b, tol)

End Sub

Function simp(a As Double, b As Double) As Double

simp = (f(a) + f(a + (b - a) / 2) \* 4 + f(b)) \* (b - a) / 6

End Function

Function f(c As Double) As Double

'

'f = c ^ 1

'f = c ^ 2

'f = c ^ 3

'f = c ^ 4

'f = c ^ 6

'f = Sin(c)

'f = Exp(c)

f = Exp(-c ^ 2 / 2) / Sqr(3.14159265358979 \* 2)

End Function

Function adaptive(d As Double, e As Double, tol As Double) As Double

Dim ic, ig, ee, il, ir As Double

ic = simp(d, e)

ig = simp(d, d + (e - d) / 2) + simp(d + (e - d) / 2, e)

ee = (ig - ic) / 15

If Abs(ee) > tol Then

il = adaptive(d, d + (e - d) / 2, tol / 2)

ir = adaptive(d + (e - d) / 2, e, tol / 2)

adaptive = il + ir

Else

adaptive = ig + ee

End If

End Function